EFFECT OF NONSTEADY WALL TEMPERATURE ON FORCED-CONVECTION FILM BOILING ON A FLAT PLATE

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Heat transfer associated with film boiling in forced-convection boundary-layer flow over a flat plate whose temperature is an arbitrary function of time is studied analytically using boundary-layer theory. The governing equations are solved by a perturbation technique. The effect on heat transfer of varying the fluid properties is discussed.

Film boiling in a forced-convection boundary-layer flow has been investigated by Cess and Sparrow [1] and Ito and Nishikawa [2] for the case of pure convection. In [3] the authors analyze the effect of radiation on film boiling in a similar flow. All these studies relate to steady-state conditions. In practice, film boiling is often observed under nonsteady conditions usually associated with a change of surface temperature with time. An example is offered by the cooling of large ingots, over which a liquid flows. Here, film boiling first develops as a consequence of the large temperature difference between the surface and the medium. The surface temperature varies with time and, consequently, a nonsteady state exists. We propose to employ boundary-layer theory to investigate the heat transfer associated with film boiling under nonsteady forced-convection conditions.

Consider the laminar boundary layer on a flat plate (Fig. 1), whose surface temperature T_W is uniform at any instant of time; in this case, in order to obtain vaporization on the plate, T_W must be higher than the saturation temperature T_{sat} of the fluid. It is assumed that a continuous film of vapor flows over the plate. Here, boiling is nonsteady because the temperature of the plate T_W depends on time. Our interest in the nonsteady vapor-film flow is related to the determination of the heat transfer between the plate and the medium.

For a forced nondissipative flow with constant properties the laws of conservation of mass, momentum, and energy take the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_v\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_v}{\rho_v c_{p_v}}\cdot\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The boundary conditions of the problem are written as follows: at y = 0 (surface of plate)

$$u=0, \tag{4a}$$

$$v=0, \tag{4b}$$

$$T=T_w(t); \tag{4c}$$

at $y = \delta(x, t)$ (vapor–liquid phase interface)

$$u=U_\infty, \tag{5a}$$

$$T=T_{sat}. \tag{5b}$$

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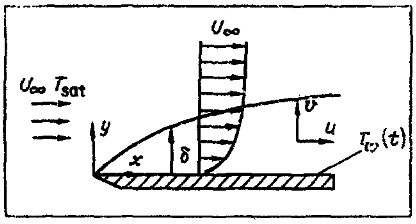

Fig. 1. Physical model and coordinate system.

On the interface at $y = \delta(x, t)$ the energy conservation condition has the form

$$\rho_v h_{fg}\left(\frac{\partial\delta}{\partial t}+u\,\frac{\partial\delta}{\partial x}-v\right)_v=-K_v\,\frac{\partial T}{\partial y}\,. \tag{6}$$

The second boundary condition, $u(\delta) = U_\infty$ (Eq. (5a)), has been used in most investigations of heat transfer associated with film boiling. However, in [1-4] this assumption was not made and it was shown that for a fluid with $c[(\rho\mu)_V/(\rho\mu)_L]^{1/2} < 0.01$, $u(\delta)$ is in fact very close to U_∞ [4].

Previous investigations of nonsteady film boiling under free convection conditions show that the film responds rapidly to the variation of the boundary condition with time [4]. Consequently, the nonsteady solution can be represented as a perturbation of the solution for the instantaneous steady state. Such a solution is most convenient when the heat transfer for a wall whose temperature depends on time is found with sufficient accuracy from quasisteady-state solutions.

Difficulties are caused by the fact that the thickness of the film is not known in advance and can be determined only as a result of solving the problem. Moreover, the thickness of the film varies along the plate and with time.

We transform system (1)-(3). We satisfy continuity equation (1), going over to the stream function

$$u=\frac{\partial\psi}{\partial y}\,,\quad v=-\,\frac{\partial\psi}{\partial x}\,. \tag{7}$$

We introduce the new dimensionless coordinate

$$\eta=\frac{y}{\delta(x,\ t)}=\frac{1}{2}\sqrt{\frac{U_\infty}{\nu x}}\;\frac{y}{\Delta(\{\lambda_n(x,\ t)\})}\,, \tag{8}$$

where

$$\{\lambda_n(x,\ t)\}=\lambda_0(x,\ t),\ \lambda_1(x,\ t),\ \ldots,\ \lambda_n(x,\ t). \tag{9}$$

Equation (8) defines η in such a way that the boundary conditions are given at $\eta = 0$ and $\eta = 1$. Δ is the dimensionless thickness of the film given by

$$\Delta\left(\{\lambda_n(x,\ t)\}\right)=\frac{1}{2}\sqrt{\frac{U_\infty}{\nu x}}\;\delta(x,\ t). \tag{10}$$

The variables $\{\lambda_n(x, t)\}$ are still arbitrary functions of x and t.

We express the dimensionless stream function and temperature as follows

$$f\left(\eta,\ \{\lambda_n(x,\ t)\}\right)=\frac{\psi(x,\ y,\ t)}{\sqrt{\nu U_\infty x}\,\Delta(\{\lambda_n(x,\ t)\})}\,, \tag{11}$$

$$\theta\left(\eta,\ \{\lambda_n(x,\ t)\}\right)=\frac{T(x,\ y,\ t)-T_{sat}}{T_w(t)-T_{sat}}\,. \tag{12}$$

In Eqs. (8), (11), and (12) we have made the following assumptions:

1) f and Θ are functions of η and $\{\lambda_n(x, t)\}$ only;

2) Δ is a function of $\{\lambda_n(x, t)\}$ only.

In the new variables the velocity components u and v take the form

$$u=\frac{U_\infty}{2}\cdot\frac{\partial f}{\partial\eta}\,, \tag{13}$$

$$v=-\frac{1}{2}\sqrt{\frac{U_\infty\nu}{x}}\,\Delta\left(f-\eta\,\frac{\partial f}{\partial\eta}\right)-\sqrt{\nu U_\infty x}\left(f-\eta\,\frac{\partial f}{\partial\eta}\right)\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x}-\sqrt{\nu U_\infty x}\,\Delta\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x}\,. \tag{14}$$

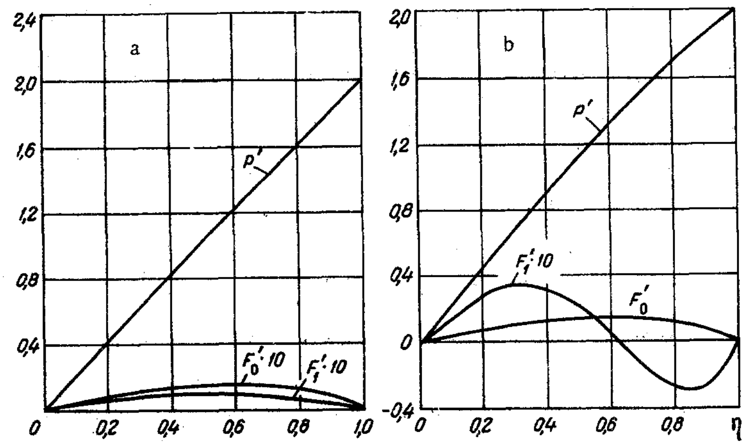

Fig. 2. Comparative velocity profiles: a) $c_p(T_w - T_{sat})/h_{fg}\mathrm{Pr} = 0.3927$; Pr = 1.0; b) $c_p(T_w - T_{sat})/h_{fg}\mathrm{Pr} = 2.2404$; Pr = 1.0.

Using (7), (8), and (12)-(14), we represent Eqs. (2) and (3) in the form

$$\frac{\partial^3 f}{\partial\eta^3} + \Delta^2 f\frac{\partial^2 f}{\partial\eta^2} = 2x\Delta^2\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial^2 f}{\partial\lambda_n\partial\eta}\cdot\frac{\partial\lambda_n}{\partial x} - 2x\Delta f\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\frac{\partial\lambda_n}{\partial x}$$

$$+ 2x\Delta^2\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x} - \frac{4\eta x\Delta}{U_\infty}\cdot\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial t} + \frac{4x\Delta^2}{U_\infty}\sum_{n=0}^{\infty}\frac{\partial^2 f}{\partial\eta\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial t}, \tag{15}$$

$$\frac{1}{\mathrm{Pr}}\cdot\frac{\partial^2\theta}{\partial\eta^2} + \Delta^2 f\frac{\partial\theta}{\partial\eta} = \frac{4x\Delta^2}{U_\infty}\sum_{n=0}^{\infty}\frac{\partial\theta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial t} + \frac{4x\Delta^2}{U_\infty}\theta\frac{1}{(T_w - T_{sat})}$$

$$\times\frac{d(T_w - T_{sat})}{dt} - \frac{4\eta x\Delta}{U_\infty}\cdot\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial t} - 2x\Delta f\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x} - 2x\Delta^2\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x}. \tag{16}$$

In Eq. (16) the expression $d(T_w - T_{sat})/dt$ represents the derivative of the wall temperature with respect to time.

The equation of conservation of energy at the interface (Eq. (6)) can be written in the following form: at $\eta = 1$

$$\frac{4x\Delta}{U_\infty}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial t} + \Delta^2 f + 2x\Delta f\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}\cdot\frac{\partial\lambda_n}{\partial x} + 2x\Delta^2\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}\frac{\partial\lambda_n}{\partial x} = -\frac{c_p(T_w - T_{sat})}{\mathrm{Pr}\,h_{fg}}\frac{\partial\theta}{\partial\eta}. \tag{17}$$

In order for Eqs. (15)-(17) to contain only functions of η and $\{\lambda_n(x, t)\}$, it is necessary that the variables x and t be eliminated from the equations in explicit form. An analysis of Eqs. (15)-(17) shows that this is the case if $\{\lambda_n(x, t)\}$ is determined from the relation

$$\lambda_n(x, t) = \left(\frac{x}{U_\infty}\right)^{n+1}\frac{1}{(T_w - T_{sat})}\cdot\frac{d^{n+1}(T_w - T_{sat})}{dt^{n+1}}, \tag{18}$$

where T_w is a continuously differentiable function of time t.

From Eqs. (15)-(17), using (18), we obtain

$$\frac{\partial^3 f}{\partial\eta^3} + \Delta^2 f\frac{\partial^2 f}{\partial\eta^2} = 2\Delta^2\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\lambda_n\frac{\partial^2 f}{\partial\lambda_n\partial\eta} - 2\Delta f\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}(n+1)\lambda_n\frac{\partial\Delta}{\partial\lambda_n} + 2\Delta^2\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}(n+1)\lambda_n\frac{\partial f}{\partial\lambda_n}$$

$$-4\eta\Delta\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}(\lambda_{n+1} - \lambda_0\lambda_n)\frac{\partial\Delta}{\partial\lambda_n} + 4\Delta^2\sum_{n=0}^{\infty}(\lambda_{n+1} - \lambda_0\lambda_n)\frac{\partial^2 f}{\partial\lambda_n\partial\eta}, \tag{19}$$

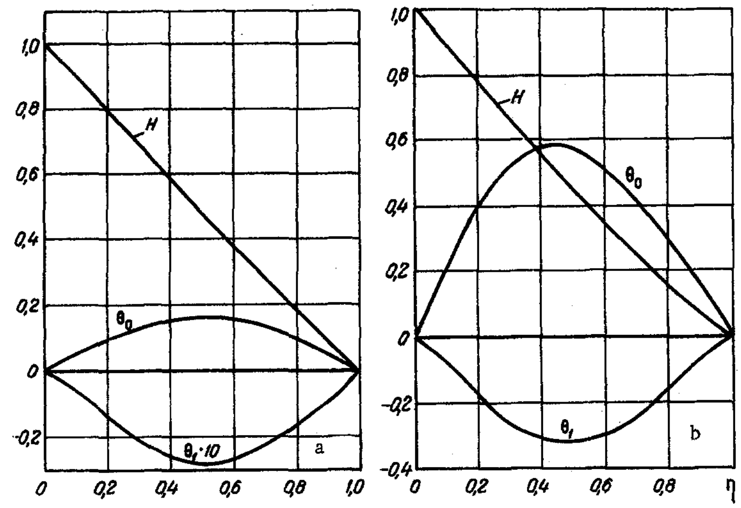

Fig. 3. Comparative temperature profiles: a) $c_p(T_w - T_{sat})/h_{fg}Pr = 0.3927$; $Pr = 1.0$; b) $c_p(T_w - T_{sat})/h_{fg}Pr = 2.2404$; $Pr = 1.0$.

$$\frac{1}{\Pr}\cdot\frac{\partial^2\theta}{\partial\eta^2}+\Delta^2 f\frac{\partial\theta}{\partial\eta}=4\Delta^2\sum_{n=0}^{\infty}\frac{\partial\theta}{\partial\lambda_n}(\lambda_{n+1}-\lambda_0\lambda_n)+4\Delta^2\theta\lambda_0$$

$$-4\eta\Delta\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}(\lambda_{n+1}-\lambda_0\lambda_n)-2\Delta f\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\lambda_n}(n+1)\lambda_n-2\Delta^2\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}(n+1)\lambda_n, \qquad (20)$$

and at $\eta = 1$

$$4\Delta\sum_{n=0}^{\infty}(\lambda_{n+1}-\lambda_0\lambda_n)\frac{\partial\Delta}{\partial\lambda_n}+\Delta^2 f+2\Delta f\sum_{n=0}^{\infty}(n+1)\lambda_n\frac{\partial\Delta}{\partial\lambda_n}$$

$$+2\Delta^2\sum_{n=0}^{\infty}\frac{\partial f}{\partial\lambda_n}(n+1)\lambda_n=-\frac{c_p(T_w-T_{\text{sat}})}{h_{fg}\Pr}\cdot\frac{\partial\theta}{\partial\eta}. \qquad (21)$$

The boundary conditions take the form: at $\eta = 0$ (surface of plate)

$$f=0,\quad \frac{\partial f}{\partial\eta}=0,\quad \theta=1; \qquad (22)$$

at $\eta = 1$ (phase interface)

$$\frac{\partial f}{\partial\eta}=2,\ \theta=0. \qquad (23)$$

Equations (19)-(23) consist of functions of η and $\{\lambda_n\}$ only. Consequently, our initial assumptions that f, Θ, and Δ are functions of η and $\{\lambda_n\}$ only were justified. However, as before, the equations are partial differential equations. In order to reduce them to differential equations, we expand the functions f, Θ, and Δ in generalized Taylor's series in stationary functions

$$f(\eta,\lambda_0,\lambda_1,\ldots,\lambda_n,\ldots)=P(\eta)+[\lambda_0F_0(\eta)+\lambda_1F_1(\eta)+\ldots]+[\lambda_0^2F_{00}(\eta)+\ldots+\lambda_0\lambda_1F_{01}(\eta)+\ldots],$$
$$\theta(\eta,\lambda_0,\lambda_1,\ldots,\lambda_n,\ldots)=H(\eta)+[\lambda_0\theta_0(\eta)+\lambda_1\theta_1(\eta)+\ldots]+[\lambda_0^2\theta_{00}(\eta+\ldots+\lambda_0\lambda_1\theta_{01}(\eta)+\ldots], \qquad (24)$$
$$\Delta(\lambda_0,\lambda_1,\ldots,\lambda_n,\ldots)=\Gamma+[\lambda_0\Delta_0+\lambda_1\Delta_1+\ldots]+[\lambda_0^2\Delta_{00}+\ldots+\lambda_0\lambda_1\Delta_{01}+\ldots].$$

On substituting these expansions in Eqs. (19)-(23) and grouping the coefficients for λ_0, λ_1, λ_2, ..., λ_n, we obtain an infinite series of perturbed equations. Below we present the zero-order equations and the first two systems of first-order perturbed equations:

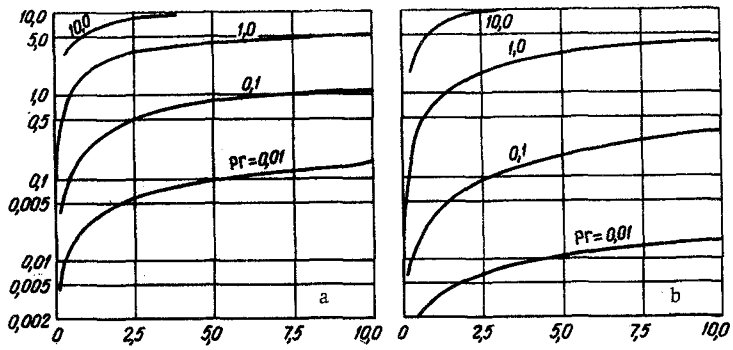

Fig. 4. Coefficient of λ_0 in Eq. (36) (a) and of λ_1 in Eq. (36) (b); abscissas: $c_p(T_w - T_{sat})/h_{fg}\mathrm{Pr}$, ordinates: $(\Theta'_{0w}/H'_w - \Delta_0/\Gamma)$ (a) and $(\Theta'_{1w}/H'_w - \Delta_1/\Gamma)$ (b).

zero order

$$P''' + \Gamma^2 PP'' = 0, \quad \frac{H''}{\mathrm{Pr}} + \Gamma^2 H'P = 0; \tag{25}$$

first system of first-order equations

$$\begin{aligned} &F_0''' + \Gamma^2 PF_0'' - 2\Gamma^2 P'F_0' - \Gamma^2 P''F_0 = -4\Gamma\Delta_0 PP'', \\ &\frac{\theta_0''}{\mathrm{Pr}} + \Gamma^2 P\theta_0' = -3\Gamma^2 F_0 H' - 4\Gamma\Delta_0 PH' + 4\Gamma^2 H; \end{aligned} \tag{26}$$

second system of first-order equations

$$\begin{aligned} &F_1''' + \Gamma^2 PF_1'' - 4\Gamma^2 P'F_1' - 3\Gamma^2 P''F_1 = -6\Gamma\Delta_1 PP'' - 4\eta\Gamma\Delta_0 P'' + 4\Gamma^2 F_0', \\ &\frac{\theta_1''}{\mathrm{Pr}} + \Gamma^2 P\theta_1' = 4\Gamma^2\theta_0 - 5\Gamma^2 F_1 H' - 6\Gamma\Delta_+ PH' - 4\eta\Gamma\Delta_0 H'. \end{aligned} \tag{27}$$

In these equations the primes denote differentiation with respect to η. The boundary conditions can now be written in the form: at $\eta = 0$

$$\begin{aligned} &P = P' = 0, \quad F_1 = F_1' = 0, \\ &F_0 = F_0' = 0, \quad H = 1, \ \theta_0 = \theta_1 = 0; \end{aligned} \tag{28}$$

at $\eta = 1$

$$P' = 2, \quad F_0' = F_1' = 0, \quad H = \theta_0 = \theta_1 = 0. \tag{29}$$

Equation (21) can be represented in the form of three equations: at $\eta = 1$

$$\Gamma^2 P = -\frac{c_p(T_w - T_{sat})}{h_{fg}\mathrm{Pr}} H', \tag{30}$$

$$3\Gamma^2 F_0 + 4\Gamma\Delta_0 P = -\frac{c_p(T_w - T_{sat})}{n_{fg}\mathrm{Pr}} \theta_0', \tag{31}$$

$$5\Gamma^2 F_1 + 6\Gamma\Delta_1 P + 2\Gamma\Delta_0 F_1 + 4\Gamma\Delta_0 = \frac{c_p(T_w - T_{sat})}{h_{fg}\mathrm{Pr}} \theta_1'. \tag{32}$$

Equations (25) with the corresponding boundary conditions are the equations of the steady state. The problem thus formulated takes the form of the fundamental problem of film boiling, the nonsteady wall temperature effects constituting a perturbation. The functions P, F_0, F_1, H, Γ, Θ_0, Θ_1, Δ_0, and Δ_1 are functions of the two parameters $c_p(T_w - T_{sat})h_{fg}$ and Pr. The general solutions of the nonsteady problem can be obtained with the aid of series (24).

In order to determine the heat transfer at the wall it is sufficient to analyze the temperature gradient $(\partial T/\partial y)_w$. From Fourier's law we have

$$q = -K_v \left(\frac{\partial T}{\partial y}\right)_w. \tag{33}$$

In the new variables Fourier's law takes the form

$$q=-\frac{K_v}{2}(T_w-T_{\text{sat}})\left(\frac{U_\infty}{\nu_v x}\right)^{1/2}\left[\frac{H_w'+\lambda_0\theta_{0w}'+\lambda_1\theta_{1w}'+\ldots}{\Gamma+\lambda_0\Delta_0+\lambda_1\Delta_1+\ldots}\right]. \tag{34}$$

If we define the heat flow in a hypothetical instantaneous steady state as

$$q_{\text{st}}=-K_v\frac{(T_w-T_{\text{sat}})}{2}\left(\frac{U_\infty}{\nu_v x}\right)^{1/2}\frac{H_w'}{\Gamma}, \tag{35}$$

we obtain the following relation

$$\frac{q}{q_{\text{st}}}=1+\lambda_0\left\{\frac{\theta_{0w}'}{H_w'}-\frac{\Delta_0}{\Gamma}\right\}+\lambda_1\left\{\frac{\theta_{1w}'}{H_w'}-\frac{\Delta_1}{\Gamma}\right\}+\ldots \tag{36}$$

From Eq. (36) we easily see that $(\Theta_{0W}'/H_W'-\Delta_0/\Gamma)$ and $(\Theta_{1W}'/H_W'-\Delta'/\Gamma)$, which henceforth will be regarded as coefficients of λ_0 and λ_1, represent the first-order nonsteady wall-temperature effect for film boiling under forced-convection conditions. Consequently, the relation between these coefficients and the physical parameters of the problem must be carefully investigated.

For the boundary conditions investigated analytic solutions of the differential equations cannot be found in closed form. Accordingly, we solved them numerically on a SDS-3600 computer. The fourth-order Runge–Kutta–Hill method, described in detail in [4], was employed.

Equations (25)-(27) were solved for Pr = 0.001; 0.01; 0.1; 1.0; and 10.0 and for Γ varying from 0.2 to 3.0. For these values it was found that $c_p(T_W-T_{sat})/h_{fg}\text{Pr}$ varies on the interval from 0.001 to 10.0.

The results are presented in Figs. 2-4. The universal constants needed to calculate the heat transfer are presented in Table 1.

The universal velocity and temperature functions are shown in Fig. 2 and Fig. 3 for Pr = 1.0 and $c_p(T_W-T_{sat})/h_{fg}$ Pr = 0.3927 and 2.2404. It is clear from these figures that the values of the higher-order terms F_0', F_1', Θ_0, Θ increase with increase in $c_p(T_W-T_{sat})/h_{fg}$. Consequently, the deviation of the velocity and temperature from the steady-state values increases with increase in superheat.

The coefficients λ_0 and λ_1 in Eq. (36) are presented in Fig. 4. It is clear from these figures that the effect of the nonsteady surface temperature T_W on heat transfer increases with increase in the Prandtl number. An increase in the Prandtl number causes either an increase in kinematic viscosity or a decrease in thermal diffusivity or both together. As may be seen from Eq. (10), the greater ν, the thicker the vapor film. An increase in the thickness of the film and a decrease in thermal diffusivity delay the thermal response of the film to a change in T_W. Consequently, the deviation of the nonsteady heat transfer from the instantaneous value in the steady state becomes greater as the Prandtl number increases.

It is clear from Fig. 4 that $(\Theta_{1W}'/H_W'-\Delta_1/\Gamma)$ is a positive quantity, whereas $(\Theta_{1W}'/H_W'-\Delta_0/\Gamma)$ is negative. Equation (18) shows that both λ_0 and λ_1 are positive when T_W' and T_W'' are positive. From Eq. (36) it is clear that the first-order effect T_W' should reduce the heat transfer as compared with the steady-state value, whereas T_W'' should increase it.

The relation between the coefficients of λ_0 and λ_1 and the parameter $c_p(T_W-T_{sat})/h_{fg}$ Pr is also clear from Fig. 4. For a given Prandtl number the coefficients of λ_0 and λ_1 increase with the increase in the parameter $c_p(T_W-T_{sat})/h_{fg}$ Pr. For a given fluid, the higher $c_p(T_W-T_{sat})/h_{fg}$ Pr, the greater the superheating (this is clear from the tabulated values of Γ). Consequently, the delay of the thermal response of the film to changes in T_W becomes longer as $c_p(T_W-T_{sat})/h_{fg}$ Pr increases.

We will determine when the heat transfer through a wall whose temperature depends on time, can be found with sufficient accuracy from the quasisteady solutions.

Case 1. Flat plate with $(T_W-T_{sat})=Mt^m$. For a flat plate whose temperature varies according to a power law, the values of λ_0 and λ_1 are calculated from Eq. (18)

$$\lambda_0=\frac{mx}{U_\infty t}$$

and

$$\lambda_1=m(m-1)\left(\frac{x}{U_\infty t}\right)^2.$$

TABLE 1. Universal Constants

Pr	$\frac{c_p(T_w - T_{sat})}{h_{fg}\,\mathrm{Pr}}$	Γ	Δ_0	Δ_1	P''_{0w}	F''_{0w}	F''_{1w}	H'_w	θ'_{0w}	θ'_{1w}
10,0	0,2352	0,4	0,2404	−0,2533	2,0269	0,0587	0,0588	−1,1368	3,0263	− 1,4128
	0,9119	0,6	0,6206	−0,8357	2,0613	0,2170	0,1421	−1,2891	6,6852	− 5,4055
	3,7030	0,8	1,0907	−1,7241	2,1108	0,4788	0,1743	−1,4864	11,9384	−13,0759
1,0	0,0397	0,2	0,0039	−0,0027	2,0066	0,0005	0,0007	−1,0133	0,0791	− 0,0012
	0,3927	0,6	0,0954	−0,0760	2,0613	0,0333	0,0359	−1,0396	0,7030	− 0,0899
	1,3376	1,0	0,3782	−0,3550	2,1771	0,1933	0,1327	−1,0952	1,9166	− 0,6017
	6,1068	1,6	1,1529	−1,2491	2,4938	0,7339	0,2197	−1,2478	4,7826	− 2,9109
	10,3331	1,8	1,4702	−1,6213	2,6454	0,9649	0,2261	−1,3210	6,0258	− 4,1550
0,1	0,1587	0,4	0,0031	−0,0021	2,0269	0,0007	0,0010	−1,0114	0,0317	− 0,0002
	0,6584	0,8	0,0231	−0,0166	2,1108	0,0101	0,0100	−1,0155	0,1264	− 0,0030
	1,5775	1,2	0,0728	−0,0521	2,2615	0,0412	0,0290	−1,0229	0,2834	− 0,0149
	3,0724	1,6	0,1624	−0,1116	2,4938	0,1034	0,0540	−1,0349	0,5015	− 0,0452
	5,4125	2,0	0,3026	−0,1961	2,8218	0,2025	0,0875	−1,0530	0,7785	− 0,1050
0,01	0,1581	0,4	0,0003	—	2,0269	—	—	−1,0102	0,0032	—
	0,6490	0,8	0,0023	−0,0016	2,1108	0,0010	0,0010	−1,0106	0,0127	—
	1,5244	1,2	0,0074	−0,0051	2,2615	0,0042	0,0031	−1,0113	0,0285	− 0,0002
	2,8773	1,6	0,0169	−0,0106	2,4938	0,0108	0,0060	−1,0126	0,0506	− 0,0005
	4,8357	2,0	0,0324	−0,0181	2,8218	0,0217	0,0103	−1,0144	0,0790	− 0,0012
0,001	0,1581	0,4	—	—	2,0269	—	—	−1,0101	0,0003	—
	1,0315	1,0	0,0004	−0,0003	2,1771	0,0002	0,0002	−1,0102	0,0020	—
	4,7824	2,00	0,0033	−0,0018	2,8218	0,0022	0,0011	−1,0105	0,0079	—

Clearly, both λ_0 and λ_1 decrease with time and at very large times become negligibly small.

In order to determine the applicability of the quasisteady solutions to problems of nonsteady heat transfer, it is necessary to investigate λ_0, λ_1 and their coefficients. We note from Fig. 4 that the maximum values of the coefficients are approximately equal to 10.0 at Pr = 10.0. If, however, λ_0 and $\lambda_1 \ll 10.0$, the nonsteady effect becomes negligibly small. For example, for linear temperature variations with respect to time (m = 1 and hence $\lambda_1 = 0$) the deviations from the instantaneous steady-state value is less than 10% for time values greater than $x/0.01\,U_\infty$ (i.e., at $t > 100\,x/U_\infty$).

Case 2. Flat plate with $(T_w - T_{sat}) = Me^{mt}$. For an exponential variation of wall temperature with time we have

$$\lambda_0 = \frac{mx}{U_\infty},$$

$$\lambda_1 = \frac{m^2x^2}{U_\infty^2}.$$

Consequently, λ_0 and λ_1 are constant with respect to time, so that the deviation of the heat-transfer rate from the instantaneous steady-state value is constant with respect to time. This deviation becomes negligibly small if λ_0 and λ_1 are much less than 0.1, which occurs at $m \ll U_\infty/10\,x$.

CONCLUSIONS

The effect of nonsteady wall temperature on the heat transfer associated with forced-convection film boiling on a flat plate has been investigated using laminar boundary-layer theory. The case of a uniform wall temperature depending arbitrarily on time is considered. The first-order deviation of the heat transfer from the instantaneous steady-state value is obtained in the form of a series in the dimensionless variables and the parameters $c_p(T_w - T_{sat})/h_{fg}$ and Pr.

On the basis of the results obtained it is possible to determine when the heat transfer for a time-dependent wall temperature can be obtained with sufficient accuracy from the quasisteady solutions. An analysis shows that:

1) the deviation of the velocity and temperature values from the steady-state values increases with increase in superheat;

2) the effect of the nonsteady wall temperature increases with increase in the Prandtl number;

3) for a given fluid the deviation of the heat-transfer rate from the instantaneous steady-state value increases with increase in superheat;

4) for an exponential time dependence of the wall temperature there is a constant deviation of the heat-transfer rate from the instantaneous steady-state value at all time values. In this case the heat-transfer rate may be considered quasisteady if $m \ll U_\infty/10\ x$;

5) when the temperature varies with time according to a power law the heat-transfer rate may be regarded as quasisteady at all values $t \gg mx/10\ U_\infty$.

NOTATION

c_p	is the specific heat of the vapor at constant pressure;
f	is the nonsteady dimensionless stream function;
F_0 and F_1	are the perturbed dimensionless stream functions;
g	is the acceleration due to gravity;
H	is the steady-state dimensionless temperature;
h_{fg}	is the latent heat of vaporization;
K	is thermal conductivity;
P	is the steady-state dimensionless stream function;
Pr	is the Prandtl number of the vapor;
q	is the local heat transfer from the wall;
T	is temperature;
t	is time;
u	is the velocity component in the x direction;
U_∞	is the free-stream velocity;
v	is the velocity component in y direction;
x	is a coordinate measuring the distance along the plate from the leading edge;
y	is a coordinate measuring the distance normal to the plate;
Γ	is the steady-state dimensionless film thickness;
Δ	is the nonsteady dimensionless film thickness;
Δ_0 and Δ_1	are perturbed dimensionless film thicknesses;
δ	is the nonsteady film thickness (dimensional);
η	is a dimensionless similarity variable (Eq. (8));
$\{\lambda_n\}$	is an infinite set of dimensionless variables (Eq. (18));
Θ	is the nonsteady dimensionless temperature;
Θ_0 and Θ_1	are perturbed dimensionless temperatures;
μ	is the absolute viscosity;
ν	is the kinematic viscosity;
ρ	is density;
ψ	is the stream function;

Subscripts

L	denotes liquid;
sat	denotes saturation;
v	denotes vapor;
w	denotes wall.

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